Radiative muon pair production in high energy electron-positron annihilation process

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(Dated: February 1, 2008)

The lowest order radiative correction to the differential cross-section of process of muon pair production with emission of hard photon at high energy electron-positron annihilation are calculated. Taking into account the emission of additional soft and hard photon the cross-section can be put in the form of Drell-Yan process in leading logarithmical approximation. Applying the crossing transformation we obtain the cross section of radiative electron-muon high-energy scattering process. Virtual and soft photon emission contributions of non-leading form are tabulated for several typical kinematical points. The limit of small invariant mass of muon pair is in agreement with our previous analysis.

I. INTRODUCTION

Process of muon pair production as well as radiative muon pair production at high energy in electron-positron collisions is commonly used for calibration purposes. One of the motivations of our investigation is the high theoretical accuracy required for description of differential cross-section. An additional interest appears in the case of small invariant mass of th muon pair. For this case the radiative muon pair production is provided by the initial state hard photon emission kinematics. It can be used as a calibration process in studying the hadronic systems of small invariant masses created by virtual photon. The lowest order radiative corrections (RC) in that kinematics to Born cross-section as well as the leading logarithmic (LL) and next-to-leading (NL) contributions in all orders of perturbation theory (PT) were considered in our recent paper [4].

Besides the practical applications, we pursue the another aim in this paper. The problem is to check the validity renormalization group (RG) predictions concerning hard processes of type $2 \to 3$.

Basing on exact (with power accuracy $O(M_\mu^2/s)$) calculations we confirm the Drell-Yan form of the cross-section of radiative muon pair production in LL. Estimation of non-leading contributions for several kinematics points are given as well.

In conclusion we put the cross-section for crossing processes: radiative electron-muon scattering and muon pair production by photon on electron in LL.

II. BORN CROSS-SECTION AND RC

In this paper for the process

$$e^+(p_+) + e^-(p_-) \to \mu^+(q_+) + \mu^-(q_-) + \gamma(k_1) \tag{1}$$

we use the following kinematics:

$$\begin{gathered}\chi_\pm = 2k_1 p_\pm, \quad \chi'_\pm = 2k_1 q_\pm, \quad s = (p_- + p_+)^2, \quad s_1 = (q_- + q_+)^2, \\ t = (p_- - q_-)^2, \quad t_1 = (p_+ - q_+)^2, \quad u = (p_- - q_+)^2, \quad u_1 = (p_+ - q_-)^2, \\ p_\pm^2 = m^2, \quad q_\pm = M^2,\end{gathered} \tag{2}$$

where $M(m)$ is muon (electron) mass. Here all kinematical invariants are much larger than muon (electron) mass, but we take into account terms of order $\ln(M/m)$:

$$\begin{gathered}s_1 \sim s \sim -t \sim -t_1 \sim -u \sim -u_1 \sim \chi_\pm \gg M^2 \gg m^2, \\ s + s_1 + t + t_1 + u + u_1 = 0.\end{gathered} \tag{3}$$

The differential cross-section of the process with the lowest order radiative correction (RC) has the form:

$$\begin{gathered}\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Gamma} = \frac{\alpha^3}{2\pi^2 s} m_0 \left[1 + \frac{\alpha}{\pi}(\Delta_{vac} + \Delta_{ff} + \Delta_{vert} + \Delta_{box} + \Delta_{soft})\right], \\ \mathrm{d}\Gamma = \frac{\mathrm{d}^3 q_+ \mathrm{d}^3 q_- \mathrm{d}^3 k_1}{\varepsilon_+ \varepsilon_- \omega_1} \delta^4(p_+ + p_- - q_+ - q_- - k_1).\end{gathered} \tag{4}$$

It's convenient to separate starting from Born level definite contributions from hard photon emission by electron, muon block and their interference:

$$m_0 = m_0^e + m_0^\mu + m_0^{int} \tag{5}$$

where [1]

$$m_0^e = A\frac{s}{\chi_-\chi_+}, \quad m_0^\mu = A\frac{s_1}{\chi_-'\chi_+'}, \quad m_0^{int} = A\left[-\frac{t}{\chi_-\chi_-'} - \frac{t_1}{\chi_+\chi_+'} + \frac{u_1}{\chi_+\chi_-'} + \frac{u}{\chi_-\chi_+'}\right], \tag{6}$$
$$A = \frac{t^2+t_1^2+u^2+u_1^2}{ss_1}.$$

The standard evaluation of additional soft photon emission contribution gives:

$$\frac{\mathrm{d}\sigma_{soft}}{\mathrm{d}\sigma_0} = -\frac{\alpha}{4\pi^2}\int\frac{\mathrm{d}^3k_2}{\omega_2}\left(-\frac{p_-}{p_-k_2}+\frac{p_+}{p_+k_2}+\frac{q_-}{q_-k_2}-\frac{q_+}{q_+k_2}\right)^2\Bigg|_{\omega_2<\Delta\varepsilon\ll\varepsilon} \tag{7}$$
$$= \frac{\alpha}{\pi}(\Delta_s^e+\Delta_s^\mu+\Delta_s^{int}) = \frac{\alpha}{\pi}\Delta_{soft}.$$

Here we denote:

$$\Delta_s^e = 2(\rho_s+L-1)\ln\frac{m\Delta\varepsilon}{\lambda\varepsilon}+\frac{1}{2}(\rho_s+L)^2-\frac{\pi^2}{3}, \tag{8}$$
$$\Delta_s^\mu = 2(\rho_{s_1}-L-1)\ln\frac{M\Delta\varepsilon}{\lambda\sqrt{\varepsilon_+\varepsilon_-}}+\frac{1}{2}(\rho_{s_1}-L)^2-\frac{1}{2}\ln^2\frac{\varepsilon_+}{\varepsilon_-}-\frac{\pi^2}{3}+\mathrm{Li}_2\left(\frac{1+c}{2}\right),$$
$$\Delta_s^{int} = \frac{1}{2}(\rho_{t_1}+\rho_u)\ln\frac{t_1}{u}+\frac{1}{2}(\rho_t+\rho_{u_1})\ln\frac{t}{u_1}+2\ln\frac{t_1}{u}\ln\frac{\sqrt{mM}\Delta\varepsilon}{\lambda\sqrt{\varepsilon\varepsilon_+}}+2\ln\frac{t}{u_1}\ln\frac{\sqrt{mM}\Delta\varepsilon}{\lambda\sqrt{\varepsilon\varepsilon_-}}$$
$$+\mathrm{Li}_2\left(\frac{1+c_-}{2}\right)+\mathrm{Li}_2\left(\frac{1-c_+}{2}\right)-\mathrm{Li}_2\left(\frac{1+c_+}{2}\right)-\mathrm{Li}_2\left(\frac{1-c_-}{2}\right),$$

where

$$L=\ln\frac{M}{m}, \quad \rho_\lambda=\ln\frac{mM}{\lambda^2}, \quad \rho_s=\ln\frac{s}{mM} \quad \rho_{s_1}=\ln\frac{s_1}{mM}, \quad \rho_t=\ln\frac{-t}{mM}, \tag{9}$$
$$\rho_{t_1}=\ln\frac{-t_1}{mM}, \quad \rho_u=\ln\frac{-u}{mM}, \quad \rho_{u_1}=\ln\frac{-u_1}{mM}, \quad \mathrm{Li}_2(z)=-\int\limits_0^z\frac{\mathrm{d}x}{x}\ln(1-x),$$
$$c_\pm=\cos(\vec{p}_-\vec{q}_\pm), \quad c=\cos(\vec{q}_+\vec{q}_-).,$$

and $\varepsilon, \varepsilon_\pm$ are the energies (in cms) of electron, muon and λ is "photon mass".

Let us now consider RC arising from the Dirac form factor of leptons and vacuum polarization, (the Pauli form factor contribution is suppressed in our kinematics). They are:

$$\Delta_{ff}+\Delta_{vac}=\frac{2m_0^e+m_0^{int}}{m_0}\left(\mathrm{Re}\Gamma(\frac{s_1}{M^2})+\mathrm{Re}\Pi(s_1)\right)+\frac{2m_0^\mu+m_0^{int}}{m_0}\left(\mathrm{Re}\Gamma(\frac{s}{m^2})+\mathrm{Re}\Pi(s)\right) \tag{10}$$

with

$$\mathrm{Re}\Gamma(\frac{s}{m^2})=(\ln\frac{m}{\lambda}-1)(1-\rho_s-L)-\frac{1}{4}(\rho_s+L)^2-\frac{1}{4}(\rho_s+L)+\frac{\pi^2}{3}, \tag{11}$$
$$\mathrm{Re}\Gamma(\frac{s_1}{M^2})=(\ln\frac{M}{\lambda}-1)(1-\rho_{s_1}+L)-\frac{1}{4}(\rho_{s_1}-L)^2-\frac{1}{4}(\rho_{s_1}-L)+\frac{\pi^2}{3},$$
$$\mathrm{Re}\Pi(s_i)=\mathrm{Re}\Pi^e(s_j)+\mathrm{Re}\Pi^\mu(s_j)+\mathrm{Re}\Pi^\tau(s_j)+\mathrm{Re}\Pi^h(s_j),$$
$$\mathrm{Re}\Pi^e(s_j)=\frac{1}{3}(\rho_{s_j}+L)-\frac{5}{9}, \quad \mathrm{Re}\Pi^\mu(s_j)=\frac{1}{3}(\rho_{s_j}-L)-\frac{5}{9}.$$

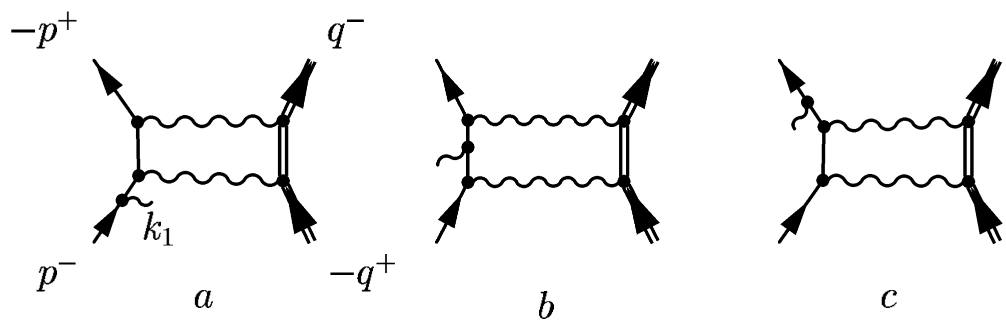

Fig. 1: Set of box-type FD used in calculation.

Here s_j is the kinematical invariant s or s_1. The contributions from the vacuum polarization from the heavy lepton τ and hadrons Π^τ, Π^h are given in [9].

III. CALCULATIONS OF BOX-TYPE RC

Consider now amplitudes arising from box-type Feynman Diagrams (FD). There are twelve FD of such a kind, or 48 squared matrix elements. In calculation we restrict ourselves by consideration of only three of box-type FD. Really the total contribution of interference of box-type and Born amplitudes can be expressed in form:

$$\mathrm{Re}\Sigma M_{box} M_0^\star = (1+P_1)[(1-P_2)B^e(M_0^e)^* + (1+P_2)B^e(M_0^\mu)^*], \tag{12}$$

with $M_0^e + M_0^\mu = M_0$, $M_0^e(M_0^\mu)$-are electron (muon) block emission part of the Born matrix element; B^e-is the electron emission part of contribution to the box-type amplitude with uncrossed photon legs (see Fig.1). Note that calculating the B^e we must consider the pentagon type FD (see Fig.1,b) and two remaining ones (see FD Fig.1a,c).

The substitution operators $P_{1,2}$ work as

$$\begin{aligned} P_1 f(p_+, p_-; q_+, q_-, k_1) &= f(q_+, q_-; p_+, p_-; -k_1); \\ P_2 f(p_+, p_-; q_+, q_-, k_1) &= f(p_+, p_-; q_-, q_+, k_1). \end{aligned} \tag{13}$$

The operator P_1 "changes" the photon emission from electron line to muon line. The application of operator P_2 permits to obtain the contribution from FD at Fig1 FD with crossed virtual photon lines. As a result we obtain:

$$\Delta_{box} = -(\rho_s + \rho_\lambda) \ln \frac{tt_1}{uu_1} + \Delta_B^{NL}. \tag{14}$$

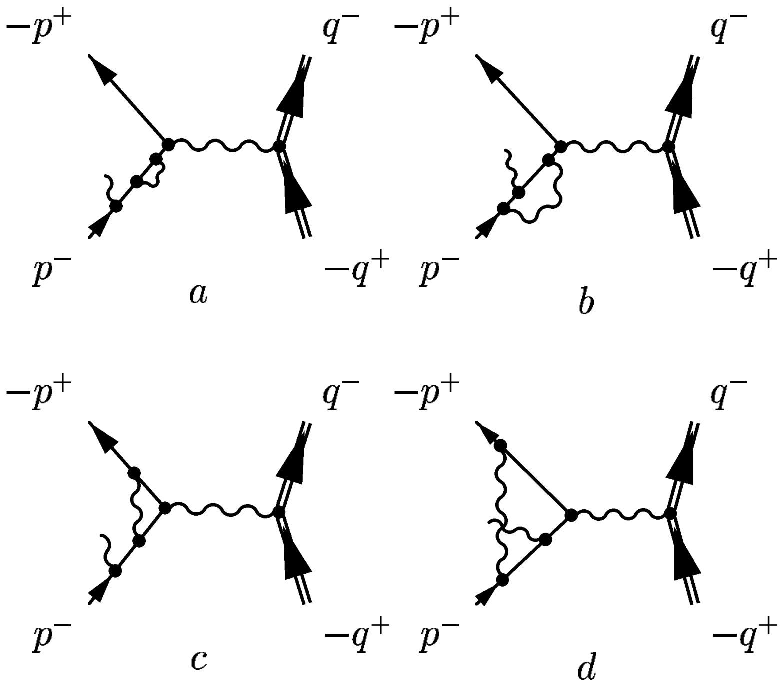

Fig. 2: Set of vertex FD used in our calculation.

The expression for Δ_B^{NL} is rather cumbersome. The whole contribution to Δ_{NL} (which does not contains large logarithms) would be given in form of the table below.

IV. VERTEX-TYPE FD

Let now consider the contribution arising from FD with vertex type insertions V^e (see Fig.2). The other vertex contributions appear from this ones by using substitutions.

$$\mathrm{Re}\Sigma M_{vertex} M_0^\star = (1+P_1)(1+P_3)V^e(M_0^e)^*, \tag{15}$$

with operator P_3 defined as:

$$P_3 f(p_+, p_-; q_+, q_-, k_1) = f(p_-, p_+, q_+, q_-;, k_1). \tag{16}$$

The total answer for vertex-type contribution reads:

$$\Delta_{vert} = -\frac{1}{2}\frac{m_e + \frac{1}{2}m_i}{m_0}[(\rho_s + L)^2 + 2(\rho_s + L)(\rho_\lambda + L) - 3(\rho_s + L) + \Delta_v^{NL}(s)]$$
$$-\frac{1}{2}\frac{m_\mu + \frac{1}{2}m_i}{m_0}[(\rho_{s_1} - L)^2 + 2(\rho_{s_1} - L)(\rho_\lambda - L) - 3(\rho_{s_1} - L) + \Delta_v^{NL}(s_1)]. \tag{17}$$

V. MASTER-FORMULA

Extracting the explicate dependence on vacuum polarization in the form $\frac{1}{|1-\Pi|^2}$ and collecting the leading and non-leading terms arising from soft photon emission, vertex and box-type FD contributions, as well as lepton form-factors we arrive to the formula:

$$\Delta_{soft} + \Delta_{box} + \Delta_{vert} + \Delta_{ff} = \Delta_{lead} + \Delta_{NL}. \tag{18}$$

This expression is free from the infrared singularities as well as from squares of large logarithms. The form of Δ_{lead} is consistent with renormalization group prescriptions:

$$1 + \frac{\alpha}{\pi}\Delta_{lead} = (1 + \frac{\alpha}{2\pi}\ln\frac{s}{m_e^2}P_\Delta(\varepsilon))^2(1 + \frac{\alpha}{2\pi}\ln\frac{s_1}{M^2}P_\Delta(\varepsilon_+))(1 + \frac{\alpha}{2\pi}\ln\frac{s_1}{M^2}P_\Delta(\varepsilon_-)) + \mathrm{O}(\alpha^2), \tag{19}$$

with P_Δ being the δ part of the kernel of evolution equation:

$$P_\Delta(\varepsilon) = 2\ln\frac{\Delta\varepsilon}{\varepsilon} + \frac{3}{2},$$
$$P_\Delta(\varepsilon_\pm) = 2\ln\frac{\Delta\varepsilon}{\varepsilon_\pm} + \frac{3}{2}. \tag{20}$$

An additional hard photon emission contribution in leading logarithmical order can be taken into account using the quasi-real electron's method [8]. It results in the replacement P_Δ by the whole kernel of evolution equation of twist 2 operators

$$P(z) = P^{(1)}(z) = \lim_{\Delta\to 0}[P_\Delta\delta(1-z) + P_\Theta(z)],$$
$$P_\Delta = 2\ln\Delta + \frac{3}{2}, \quad P_\Theta(z) = \Theta(1-\Delta-z)\frac{1+z^2}{1-z}. \tag{21}$$

As a result we arrive to compact form of the cross section:

$$\frac{d\sigma^{e^+e^-\to\mu^+\mu^-\gamma}(p_-,p_+,q_-,q_+,k_1)}{d\Gamma} = \int\limits_{x_m}^{1}\mathrm{d}x_1\int\limits_{x_m}^{1}\mathrm{d}x_2\int\limits_{y_-}^{1}\frac{\mathrm{d}z_-}{z_-}\int\limits_{y_+}^{1}\frac{\mathrm{d}z_+}{z_+}D_e(x_1,s)D_e(x_2,s)\times \tag{22}$$
$$D_\mu(\frac{y_-}{z_-},s_1)D_\mu(\frac{y_+}{z_+},s_1)\frac{(1+\frac{\alpha}{\pi}K)}{|1-\Pi(sx_1x_2)|^2}\frac{\mathrm{d}\sigma^{e^+e^-\to\mu^+\mu^-\gamma}(x_1p_-,x_2p_+,Q_-,Q_+,k_1)}{d\Gamma_1},$$
$$Q_\pm = \frac{z_\pm}{y_\pm}q_\pm, \quad y_\pm = \frac{\varepsilon_\pm}{\varepsilon},$$

and the structure functions $D(x,s)$ having the standard form:

$$D_e(x,s) = \delta(1-x) + \frac{\alpha}{2\pi}P^{(1)}(x)\ln\frac{s}{m^2} + \ldots\ ,$$
$$D_\mu(y,s_1) = \delta(1-y) + \frac{\alpha}{2\pi}P^{(1)}(y)\ln\frac{s_1}{M^2} + \ldots\ . \tag{23}$$

The phase volumes entering the left and right parts of master equation are different:

$$\mathrm{d}\Gamma = \frac{\mathrm{d}^3 q_-}{\varepsilon_-}\frac{\mathrm{d}^3 q_+}{\varepsilon_+}\frac{\mathrm{d}^3 k_1}{\omega_1}\delta(p_+ + p_- - q_+ - q_- - k_1),$$
$$\mathrm{d}\Gamma_1 = \frac{\mathrm{d}^3 Q_-}{E_-}\frac{\mathrm{d}^3 Q_+}{E_+}\frac{\mathrm{d}^3 k_1}{\omega_1}\delta(x_2 p_+ + x_1 p_- - Q_+ - Q_- - k_1), \qquad (24)$$
$$E_\pm = \frac{z_\pm}{y_\pm}\varepsilon_\pm .$$

The lower limits of the energy fractions integrations x_m, y_m are determined by the experiment set-up conditions. The quantity K (so called K-factor) collects all the nonleading contributions. It has contributions from virtual, soft and hard photon emission terms. In the Table below we give its value for typical experimental points of the considered process keeping all contributions except ones arising from additional hard photon emission.

VI. CONCLUSION

Our consideration was devoted to the lowest order RC. Nevertheless result obtained reveals the lowest order expansion of the structure functions D. So the general Drell-Yan form of cross section is established, which is valid in all orders of PT. The order of magnitude of nonleading terms can be estimated from the Table 1:

N	e_-	e_+	c_-	c_+	Δ_{NL}
1	0.59	0.66	0.29	-0.06	6.77
2	0.67	0.67	0.50	0.30	3.24
3	0.68	0.65	0.69	-0.50	8.68
4	0.59	0.56	-0.30	-0.30	8.35

Table 1. Numerical estimation of Δ_{NL}, part of K-factor excluding non-leading terms arising from hard non-collinear photon emission (which depends on experimental set-ups) and the terms proportional to $\ln \Delta\varepsilon/\varepsilon$, $\ln \Delta\varepsilon/\varepsilon_\pm$ arising from soft photon emission.

Without additional calculations we can obtain by the analogy with the result given above the cross section of crossing process - radiative electron-muon scattering:

$$e_-(p_1) + \mu_-(q_1) \to e_-(p_2) + \mu_-(q_2) + \gamma(k_1) + (\gamma). \qquad (25)$$

It can be constructed in complete analogy with the Drell-Yan form of cross section of above considered process $e_+e_- \to \mu_+\mu_-\gamma$, using in right hand side as a hard subprocess the Born cross section:

$$\frac{d\sigma_B^{e\mu\gamma}}{d\Gamma_{e\mu\gamma}}(p_1,q_1;p_2,q_2,k_1)=\frac{\alpha^3}{16\pi^2(p_1q_1)}\frac{(p_1q_2)^2+(p_1q_1)^2+(p_2q_1)^2+(p_2q_2)^2}{(p_1p_2)(q_1q_2)}W, \tag{26}$$

with

$$d\Gamma_{e\mu\gamma}=\frac{d^3q_2}{q_{20}}\frac{d^3p_2}{p_{20}}\frac{d^3k_1}{\omega_1}\delta^4(p_1+q_1-p_2-q_2-k_1);$$
$$W=-\left(\frac{p_1}{p_1k_1}+\frac{q_1}{q_1k_1}-\frac{p_2}{p_2k_1}-\frac{q_2}{q_2k_1}\right)^2. \tag{27}$$

It is worth to note that the value of K-factor for the last process is not known.

All the 1-loop integrals used of scalar,vector and tensor types were published in our previous papers [2]. It's important to note that numerical values of nonleading terms for process of radiative muon pair production for the case of small muon invariant mass we find completely in agreement with the result of our paper devoted to this kinematical situation [4], where it was calculated analytically.

VII. ACKNOWLEDGEMENT

We are grateful to grant RFFI 03-02-17077 and two of us E.K. and B.V.to INTAS 00-00-366.

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